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A JELS Probabilistic Inventory Model for One Vendor Multi Customers Situation with Random Demands having Weibull Distribution

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Abstract : We consider a one-vendor multi-buyer integrated inventory model. The vendor seeks to minimize his total annual cost subject to the maximum costs which buyers are prepared to incur. In order to implement this model, the vendor only needs to know buyer's annual demand and previous order frequency, which can be inferred from buyer's past ordering behavior. We find the optimal solution for the one-vendor one-buyer case, and present a model using Weibulls Distribution for the one-vendor multi-buyer case. The effective ways for a compromise between the vender and multiple customers at a common lot size with certain amount of price adjustments are determined and the methodology is explained through a numerical example.

Keywords : Joint Economic Lot Size, Weibulls Distribution, Inventory Management, Economic Order Quantity, Optimal Lot Size, Operations Management, Random Inventory Stock Level, Optimum Common Lot Size, Total Relevant Cost, Absolute Cost.

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1 Introduction

A JELS probabilistic model developed for a single vendor multi customers situation where demand of the customers and stock level of the vendor are identically distributed random variables belonging to Weibulls Distribution.

A stochastic model differs substantially from multi-customer policies of Lu Lu [39,(1995)] and Drenzer and Wesolowsky [13,(1989)]. But here technique of negotiation and pricing policies have been derived from the deterministic model of Banerji A[3, (1986)]. In this paper the model has been developed and has been illustrated through a numerical example.

Coordination between two different business entities is an important way to gain competitive advantage as it lowers supply chain cost. This paper reviews literature dealing with buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. In a typical purchasing situation, the issues of price, lot sizing etc., usually are settled through negotiations between the purchaser and the vender. The effective ways for a compromise between one vender and multiple customers at a common lot size with certain amount of price adjustments are determined and the methodology is explained through a numerical example.

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In the process, such a supply chain loses to supply chain that is customer focused where the individual links orient their business processes and decisions to ensure least cost delivery of products/services to the ultimate customer. Narasimhan and Carter (1998) in their work have mentioned that a well-integrated supply chain involves coordinating the flows of materials and information between suppliers, manufacturers, and customers. Thomas and Grifin (1996) have mentioned that effective supply chain management requires planning and coordination among the various channel members including manufacturers, retailers and intermediaries if any.

Several strategies are used to align the business processes and activities of the members of a supply chain to ensure better supply chain performance in terms of cost, response time, timely supply and customer service. Supply chain coordination is concerned with the development and implementation of such strategies. There is no universal coordination strategy that will be efficient and effective for all supply chains as the performance of a coordination strategy is supply chain characteristics dependent. Supply chain coordination through quantity discount has received much attention in Production/Operations Management literature only recently (Weng, 1995a,b). Since quantity discount is considered to be one of the most popular mechanisms of coordination between the business entities, this paper primarily investigates supply chain coordination models that have used quantity discount as coordination tool under deterministic environment. However, we have also included here some integrated buyer vendor models that have similar type of objective function to achieve production distribution coordination and that improves the performance of the supply chain. In this paper, the word vendor, supplier and manufacturer is used alternatively to represent the same upstream member in the supply chain who sells the item to the buyer unless specifically mentioned.

Many researchers like Monahan [1], Lee [2], Joglekar[4], have discovered various methods of discount polices to satisfy the vendor. This paper deals with a discount policy which causes no loss to both the parties and both are getting some benefit.

2 Development of the Model

The following notation are used in developing the model.

I) i an integer such that $1 \le i \le n$.

II) C_i represents the customer i.

III) X_i = Random demand (lot size) of the customer C_i (i=1,2,3,…..n)

Where,

$$f_i(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \\ 0 & \text{otherwise} \end{cases}, \qquad 0 < x < \infty$$

if $A = \frac{k}{\lambda^k}$ then

$$I = A \int_0^\infty x^{k-1} e^{-(x/\lambda)^k} dx \quad \text{where } k > 0, \lambda > 0$$

IV) Y = The random inventory stock level (lot size) of the vendor, with density function f(y).

V) t = Scheduling time period which is a prescribed constant.

VI) C_{v1} = Carrying cost (holding cost) of the vendor per unit item per t time units.

VII) C_{v2} = Shortage cost (penalty) of the vendor per units item per t time units.

VIII) $C_{p1}^{(i)}$ = Carrying cost (holding cost) of the customer C_i per unit item per t time units.

IX) $C_{p2}^{(i)}$ = Shortage cost (penalty) of the customer C_i per unit item per t time units.

X) Z = Variable lot size (i.e., stock level or order level) which is assumed to be the same for the vendor and the individual customer during negotiation.

XI) $Z_p^{(i)^*}$ = Optimum lot size of the customer C_i .

XII) Z_v^* = Optimum lot size of the vendor.

XIII) $Z^{(i)^*}$ = Optimum common lot size when the vendor and the customer C_i adopt Joint Economic Lot Size (JELS).

XIV) $\propto^{(i)} = \frac{C_{p2}^{(i)}}{C_{v2}}$

XV) $\beta^{(i)} = \frac{C_{p1}^{(i)} + C_{p2}^{(i)}}{C_{v1} + C_{v2}}$

XVI) $C_v(Z_v^*)$ = Optimum (i.e. minimum) total relevant cost TRC of the vendor.

XVII) $C_p^{(i)}(Z_p^{(i)^*})$ = Optimum (i.e. minimum) TRC of the customer C_i.

XVIII) $C_p^{(i)}(Z_v^*)$ = Total relevant cost of the customer C_i if he adopts the optimum lot size Z_v^* of the vendor.

XIX) $C_v(Z_p^{(i)^*})$ = Total relevant cost of the vendor if he adopts the optimum lot size $Z_p^{(i)}$ of the customer C_i.

XX) $C_p^{(i)}(Z^{(i)^*})$ = The total relevant cost of the customer C_i if he adopts $Z^{(i)^*}$.

XXI) $C_v(Z^{(i)^*})$ = The total relevant cost of the vendor if he adopts $Z^{(i)^*}$.

XXII) $ACA_\theta\ (Z' \rightarrow Z'')$ = Absolute cost advantage of the party θ when the party θ changes from the lot size Z' to the lot size Z'' at any point of the time. The party may be the vendor or any individual customer C_i.

XXIII) $ACP_\theta\ (Z' \rightarrow Z'')$ = Absolute cost penalty of the party θ when the party changes from lot size Z' to the lot size Z'' at any point of time. The party may be the vendor or any individual customer C_i.

XXIV) JACA $(Z_p^{(i)^*} \rightarrow Z^{(i)^*})$ = Joint Absolute Cost Advantage during negotiation between the vendor and the individual customer where

$$JACA(Z_p^{(i)^*} \rightarrow Z^{(i)^*}) = ACA_v\,(Z_p^{(i)^*} \rightarrow Z^{(i)^*}) - ACP_p\,(Z_p^{(i)^*} \rightarrow Z^{(i)^*})$$

XXV) $E(X_i)$ = Expectation of the random lot size of the customer C_i.

XXVI) $E(Y)$ = Expectation of the random lot size of the vendor.

2.1 Assumptions

The following assumptions are used in developing the model.

(1) X_i (i=1,2,3,... n) and Y are identically distributed independent random variables belonging to Weibull distribution with density function f_1 & f . So, that $f_1(x) = f(x)$ for all $x \in R$.

(2) Initially n customers come to a vendor together and place orders to the vendor for a particular item of goods for which the vendor is the sole supplier.

(3) There is a perfect understanding between the vendor and all the customers to part with the cost information and to agree upon a common price adjustment.

(4) On receiving the cost information from the customers, the vendor calculates his own Economic Lot Size (ELS) as well as the ELS of each customer C_i (i = 1, 2, 3, … n)

(5) On the basis of the cost information received from the vendor, each customer computes his own ELS and the ELS of the vendor independently.

(6) After proper negotiation between the vendor and the individual customer, the vendor finds his optimum cost and production inventory plan and calculate a reasonable and uniform price support he may offer to the customers to satisfy all of them.

(7) While fixing the unit price discount the vendor has to estimate the joint benefit of optimization between himself and individual customer C_i by dividing the total joint benefit by the expected demand of C_i with a view to satisfy the customer.

(8) There is no setup cost.

(9) Shortages are allowed for each party (i.e. each customer C_i and the vendor.)

(10) Either the replenishment is instantaneous or the buffer stock available with the vendor is high enough to meet the total demand of the customers immediately, as soon as the negotiation is over and orders are placed.

2.2 The Model with n customers

X_i (i = 1, 2, 3. . . n) and Y are n+1 independent and identically distributed random variables belonging to Weibulls distribution,

Therefore,

$f_i(x)=f(x)= \int_0^{Ax^{k-1}e^{-(x/\lambda)^k}}_{otherwise} \quad 0 < x < \infty$

Where A = $\frac{K}{\lambda^k}$ then

$=A\int_0^{\infty} x^{k-1}e^{-(x/\lambda)^k}$ dx

where k > 0, λ > 0

The vendor negotiates with an individual customer say C_i, and a compromise is arrived at, to adopt individual JELS $Z^{(i)^*}$ with a price adjustment in the form of discount . This will be generalized to all the values of i (i = 1, 2, 3, . . . n). Then a common strategy for individual lot size and price adjustment has to be designed by the vendor.

Corresponding to the optimum value for vendor, customers, the following results can be obtained.

$Z_p^{(i)^*} = \varphi_1^{-1}[\frac{C_{p2}^{(i)}}{A(C_{p1}^{(i)}+C_{p2}^{(i)})}]$ where

$$\varphi_1(Z) = \int_0^Z x^{k-1}\, e^{-(x/\lambda)^k} dx \qquad (2.2.1)$$

$$Z_v^* = \varphi_1^{-1}[\frac{C_{v2}}{A(C_{v1}+C_{v2})}] \qquad (2.2.2)$$

$$C_p^{(i)^*}(Z_p^{(i)^*}) = [C_{p2}^{(i)}.\lambda.(\frac{1}{k})! - (C_{p1}^{(i)}+C_{p2}^{(i)})$$

$$A\varphi_2(Z_p^{(i)^*})] \text{ where } \varphi_2(Z) = \int_0^Z x^k\, e^{-(x/\lambda)^k} dx \qquad (2.2.3)$$

$$C_v^{(i)}(Z_v^*) = [C_{v2}\,\lambda\left(\frac{1}{k}\right)! - (C_{v1}+C_{v2})$$

$$A\varphi_2(Z_v^*)] \qquad (2.2.4)$$

$C_p^{(i)}(Z_v^*)$ =

$[C_{v2}\,\beta Z_v^* - (C_{p1}^{(i)}+C_{p2}^{(i)})A\varphi_2\,(Z_v^*) - C_{p2}^{(i)}Z_v^* + C_{p2}^{(i)}.\lambda.(\frac{1}{k})!)]$ (2.2.5)

$C_v(Z_p^{(i)^*}) = [\frac{Z_p^{(i)^*}\,C_{p2}^{(i)}}{\beta} - (C_{v1}+C_{v2})A\varphi_2(Z_p^{(i)^*}) - C_{v2}Z_p^{(i)^*} + C_{v2}.\lambda.(\frac{1}{k})!]$ (2.2.6)

$Z^{(i)^*} = \varphi_1^{-1}[\frac{C_{p2}^{(i)}+C_{v2}}{A(C_{p1}^{(i)}+C_{p2}^{(i)}+C_{v1}+C_{v2})}]$ (2.2.7)

$C_p^{(i)}(Z^{(i)^*})$=

$[Z^{(i)^*}\frac{(C_{p2}^{(i)}+C_{v2})\beta}{(\beta+1)} - \left(C_{p1}^{(i)}+C_{p2}^{(i)}\right)A\,\varphi_2(Z^{(i)^*}) - C_{p2}^{(i)}Z^{(i)^*} + C_{p2}^{(i)}.\lambda.(\frac{1}{k})!]$ (2.2.8)

$C_v(Z^{(i)^*})$=

$Z^{(i)^*}\frac{(C_{p2}^{(i)}+C_{v2})}{(\beta+1)} - (C_{v1}+C_{v2})A\varphi_2(Z^{(i)^*}) - C_{v2}Z^{(i)^*} + C_{v2}\lambda\left(\frac{1}{k}\right)!]$ (2.2.9)

Lemma (2.2.1)

(a). $\alpha^{(i)} < \beta^{(i)} \Rightarrow Z_p^{(i)^*} < Z^{(i)^*} < Z_v^*$ (2.2.10a)

(b). $\alpha^{(i)} > \beta^{(i)} \Rightarrow Z_p^{(i)^*} > Z^{(i)^*} > Z_v^*$ (2.2.10b)

(c). $\alpha^{(i)} = \beta^{(i)} \Rightarrow Z_p^{(i)^*} = Z^{(i)^*} = Z_v^*$ (2.2.10c)

The negotiation between the vendor and the individual customer C_i will be exactly the same as one vendor one customer situation. If the ELS of the vendor be in effect and all the customers change over to respective JELS, then this will give rise to a situation in which customers will suggest the vendor for a unit price increase. We ignore this because such type of bargain is against the current practice . Hence ignoring such a possibility we concentrate upon the ELS of individual customer in effect trying to switch over to the individual JELS $Z^{(i)^*}$.

Lemma (2.2.2)

If $\alpha^{(i)} \neq \beta^{(i)}$ *,then*

(i). $C_v(Z_p^{(i)^*}) - C_v(Z^{(i)^*}) > 0$

(ii). $C_p(Z^{(i)^*}) - C_p(Z_p^{(i)^*}) > 0$

2.3 ELS of customers in effect $(Z_p^{(i)^*} \to Z^{(i)^*})$

Let both the vendor and the i[th] customer adopt JELS $Z^{(i)^*}$, when i[th] customer's lot size $Z_p^{(i)^*}$ is in effect. This adoption will be done separately for individual customers. That is each C_i will adopt joint lot size $Z^{(i)^*}$ when $Z_p^{(i)^*}$ is in effect $(i = 1, 2, 3, \ldots n)$ we have , by this type of adoption the vendor will be at an advantageous position and the i[th] customer will be at a loss i = 1, 2, 3, . . . n. Now we calculate the difference between the absolute cost advantage of the vendor and the absolute cost penalty of the i[th] customer.

As,

$ACA_v\,(Z_p^{(i)^*} \to Z^{(i)^*}) > ACP_p\,(Z_p^{(i)^*} \to Z^{(i)^*})$

Hence,

$JACA_v(Z_p^{(i)^*} \to Z^{(i)^*}) = ACA_v\ Z_p^{(i)^*} \to Z^{(i)^*}) - ACA_p(Z_p^{(i)^*} \to Z^{(i)^*}) > 0$

Now $ACA_v(Z_p^{(i)^*} \to Z^{(i)^*}) = C_v(Z_p^{(i)^*}) - C_v(Z^{(i)^*})$ from (2.2.6) & (2.2.9)

$= [\frac{Z_p^{(i)^*} C_{p2}^{(i)}}{\beta} - (C_{v1} + C_{v2}) A\varphi_2(Z_p^{(i)^*} - C_{v2} Z_p^{(i)^*}$

$+ C_{v2}\lambda.(\frac{1}{k})!] - [Z^{(i)^*} \frac{(C_{p2}^{(i)} + C_{v2})}{(\beta+1)} -$

$(C_{v1} + C_{v2}) A\varphi_2(Z^{(i)^*}) -$

$C_{v2} Z^{(i)^*} + C_{v2}\lambda\left(\frac{1}{k}\right)!]$

$= [\frac{(C_{p2}^{(i)} + Z_p^{(i)^*})}{\beta} - (C_{v1} + C_{v2}) A\varphi_2(Z_p^{(i)^*})$

$- C_{v2} Z_p^{(i)^*} + C_{v2}\lambda\left(\frac{1}{k}\right)! - Z^{(i)^*}\frac{(C_{p2}^{(i)} + C_{v2})}{(\beta+1)} +$

$(C_{v1} + C_{v2}) A\varphi_2 Z^{(i)^*} + C_{v2} Z^{(i)^*} -$

$C_{v2}\lambda\left(\frac{1}{k}\right)!]$ (2.3.1)

And

$ACA_{ci}(Z_p^{(i)^*} \rightarrow Z^{(i)^*}) = C_p^{(i)}(Z^{(i)^*}) - C_p^{(i)}(Z_p^{(i)^*})$

$= [Z^{(i)^*}\frac{(C_{p2}^{(i)} + C_{v2})\beta}{(\beta+1)} - (C_{p1}^{(i)} + C_{p2}^{(i)}) A\,\varphi_2(Z^{(i)^*})$

$- C_{p2}^{(i)} Z^{(i)^*} + C_{p2}^{(i)}\lambda\left(\frac{1}{k}\right)!] - [C_{p2}^{(i)}\lambda\left(\frac{1}{k}\right)!$

$-(C_{p1}^{(i)} + C_{p2}^{(i)}) A\,\varphi 2(Z_p^{(i)^*})]$

$= [\frac{(C_{p2}^{(i)} + C_{v2})\beta}{(\beta+1)} - (C_{p1}^{(i)} + C_{p2}^{(i)}) A\,\varphi_2(Z^{(i)^*})$

$- C_{p2}^{(i)} Z^{(i)^*} + C_{p2}^{(i)}\lambda\left(\frac{1}{k}\right)! - C_{p2}^{(i)}\lambda\left(\frac{1}{k}\right)! +$

$(C_{p1}^{(i)} + C_{p2}^{(i)}) A\,\varphi_2(Z_p^{(i)^*})]$

$= [\frac{(C_{p2}^{(i)} + C_{v2}) Z^{(i)^*}\beta}{(\beta+1)} - (C_{p1}^{(i)} + C_{p2}^{(i)}) A\,(\varphi_2(Z^{(i)^*})$

$- \varphi_2(Z_p^{(i)^*})) - C_{p2}^{(i)} Z^{(i)^*}]$ (2.3. 2)

Therefore,

$JACA(Z_p^{(i)^*} \rightarrow Z^{(i)^*}) =$

$ACA_v(Z_p^{(i)^*} \rightarrow Z^{(i)^*}) - ACP_p(Z_p^{(i)^*} \rightarrow Z^{(i)^*})$

$= [\frac{C_{p2}^{(i)} Z_p^{(i)^*}}{\beta} - Z^{(i)^*}\frac{(C_{p2}^{(i)} + C_{v2})}{\beta+1} + (C_{v1} + C_{v2})$

$A(\varphi_2(Z)^{(i)^*} - \varphi_2 Z_p^{(i)^*}) + C_{v2}(Z^{(i)^*} - Z_p^{(i)^*})]$

$= [\frac{(C_{p2}^{(i)} + C_{v2}) Z^{(i)^*}\beta}{(\beta+1)} - (C_{p1}^{(i)} + C_{p2}^{(i)}) A\,(\varphi_2(Z_p^{(i)^*})$

$- \varphi_2(Z^{(i)^*})) - C_{p2}^{(i)} Z^{(i)^*}]$

$= [\frac{C_{p2}^{(i)} Z_p^{(i)^*}}{\beta} - \frac{Z^{(i)^*} C_{p2}^{(i)}}{\beta+1} - \frac{Z^{(i)^*} C_{v2}}{\beta+1} + (C_{v1} + C_{v2})$

$A(\varphi_2 Z^{(i)^*} - \varphi_2 Z_p^{(i)^*}) + C_{v2}(Z^{(i)^*} - Z_p^{(i)^*}) -$

$\frac{C_{p2}^{(i)} Z^{(i)^*}\beta}{\beta+1} - \frac{C_{v2} Z^{(i)^*}\beta}{\beta+1} - (C_{p1}^{(i)} + C_{p2}^{(i)})$

$A(\varphi_2(Z_p^{(i)^*}) - \varphi_2(Z^{(i)^*})) + C_{p2}^{(i)} Z^{(i)^*}]$

$= [\frac{C_{p2}^{(i)} Z_p^{(i)^*}}{\beta} - \frac{Z^{(i)^*} C_{p2}^{(i)}}{\beta+1} - \frac{C_{p2}^{(i)} Z^{(i)^*}\beta}{\beta+1} +$

$C_{p2}^{(i)} Z^{(i)^*} - \frac{C_{v2} Z^{(i)^*}}{\beta+1} - \frac{C_{v2} Z^{(i)^*}\beta}{\beta+1} +$

$C_{v2}(Z^{(i)^*} - Z_p^{(i)^*}) -$

$(C_{p1}^{(i)} + C_{p2}^{(i)})\ A(\varphi_2(Z_p^{(i)^*} - \varphi_2(Z^{(i)^*})) + (C_{v1}$

$+ C_{v2}) A(\varphi_2(Z^{(i)^*}) - \varphi_2(Z_p^{(i)^*})]$

=[

$\frac{C_{p2}^{(i)} Z_p^{(i)^*}}{\beta} + Z^{(i)^*} C_{p2}^{(i)}\left(1 - \frac{1}{\beta+1} - \frac{\beta}{\beta+1}\right) - \frac{C_{v2} Z^{(i)^*}}{\beta+1}(1+\beta)$

$+C_{v2}(Z^{(i)^*} - Z_p^{(i)^*}) + (C_{p1}^{(i)} + C_{p2}^{(i)})$

$A(\varphi_2(Z^{(i)^*}) - \varphi_2(Z_p^{(i)^*})) + (C_{v1} + C_{v2})$

$A(\varphi_2(Z^{(i)^*}) - \varphi_2(Z_p^{(i)^*})]$

=

$[\frac{C_{p2}^{(i)} Z_p^{(i)^*}}{\beta} - C_{v2} Z^{(i)^*} + C_{v2} Z^{(i)^*} - C_{v2} Z_p^{(i)^*}$

$+ (C_{p1}^{(i)} + C_{p2}^{(i)} + C_{v1} + C_{v2})$

$A\{\varphi_2(Z^{(i)^*}) - \varphi_2(Z_p^{(i)^*})\}]$

$$= [C_{p2}^{(i)} Z_p^{(i)^*} - C_{v2} Z_p^{(i)^*} + (C_{p1}^{(i)} + C_{p2}^{(i)} + C_{v1} + C_{v2})A\{\varphi_2(Z^{(i)^*}) - \varphi_2(Z_p^{(i)^*})\}] \quad (2.3.4)$$

So, the optimum value of the unit price discount offered by the vendor to the customer C_i can be expressed as

$$d_{opt} = \frac{1}{2} \frac{JACA(Z_p^{(i)^*} \to Z^{(i)^*})}{E(x)}$$

$$= \frac{1}{2} \frac{JACA(Z_p^{(i)^*} \to Z^{(i)^*})}{\lambda.\left(\frac{1}{k}\right)!}$$

$$= \frac{1}{2\lambda.\left(\frac{1}{k}\right)!} JACA\,(Z_p^{(i)^*} \to Z^{(i)^*}) \quad (2.3.5)$$

Where JACA $(Z_p^{(i)^*} \to Z^{(i)^*})$ can be evaluated by using the formula given in step (2.3.4).

Now we shall extend the process to all the n customers we divide the n customers into three categories depending upon the relation between $\propto^{(i)}$ and $\beta^{(i)}$.

(i) First category : $\propto^{(i)} = \beta^{(i)}$ (i=1, 2, . . . n) say

The customer for which $\propto^{(i)} = \beta^{(i)}$, will have no difficulty in compromise between the customers and the vendor. Since for $\propto^{(i)} = \beta^{(i)}$ we have by (2.2.10c) $Z_p^{(i)^*} = Z^{(i)^*} = Z_v^*$. Hence the vendor will have no objection in fulfilling the optimum demand of that particular customer because that is also the estimated optimum lot size of the vendor.

(ii) Second Category : $\propto^{(i)} < \beta^{(i)}$

Let $\propto^{(i)} = \beta^{(i)}$ for r customers, that is $\propto^{(i)} < \beta^{(i)}$, for i = m+1, m+2, . . . m+r.
In such a situation the vendor will be at an advantageous position and these m+r customers are bound to incur loss. Hence, the vendor will have to give unit price discounts to the customers C_{m+1}, C_{m+2}, . . . Cm+r respectively as follows.

$\frac{1}{2.\lambda.\left(\frac{1}{k}\right)!}$ JACA$(Z_p^{(m+1)^*} \to Z^{(m+1)^*})$. . . .

$\frac{1}{2.\lambda.\left(\frac{1}{k}\right)!}$ JACA$(Z_p^{(m+r)^*} \to Z^{(m+r)^*})$

Where i = m+1, m+2 . . . m+r

(Since in case of Weibulls distribution E(x) $= \lambda.\left(\frac{1}{k}\right)!$)

Let

$$\delta = \text{Max} \frac{1}{2.\lambda.\left(\frac{1}{k}\right)!} JACA(Z_p^{(m+k)^*} \to Z^{(m+k)^*})$$

for $1 \leq K \leq r$ (2.3.6)

Since δ is the maximum of the unit price discounts mentioned in (2.3.6), therefore the vendor can satisfy all the customers C_{m+1}, C_{m+2}, . . C_{m+r} with this discount and make them agree to adopt their respective individual JELS instead of their original ELS.

(iii). Third Category : $\propto^{(i)} > \beta^{(i)}$

Let $\propto^{(i)} < \beta^{(i)}$ for rest of the customers, that is for I = (m+r)+1, . . . n.

$\Rightarrow Z_v^* < Z^{(i)^*} < Z_p^{(i)^*}$ for i = (m+r)+1, . . . n

Let m+r = M

$\therefore Z_v^* < Z^{(i)^*} < Z_p^{(i)^*}$ for i = M+1, M+2, . . n

So, in this case also the vendor is at an advantageous position and the customers are at a disadvantageous position as $Z_p^{(i)^*} \rightarrow Z^{(i)^*}$. Hence vendor will give unit price discounts to the customers, C_{M+1}, C_{M+2}, . . . C_n respectively as follows.

$\frac{1}{2.\lambda.\left(\frac{1}{k}\right)!}$ JACA($Z_p^{(m+k)^*} \rightarrow Z^{(m+k)^*}$). . . .

$\frac{1}{2.\lambda.\left(\frac{1}{k}\right)!}$ JACA($Z_p^{(n)^*} \rightarrow Z^{(n)^*}$)

Let $\bar{\delta}$ = Max $\frac{1}{2.\lambda.\left(\frac{1}{k}\right)!}$ JACA($Z_p^{(i)^*} \rightarrow Z^{(i)^*}$)

where i = M+1, M+2, . . . n

This $\bar{\delta}$ is the maximum unit price discount which will satisfy all the customers C_{m+r+1}, C_{m+r+2}, . . . $C_{n.}$

Let Δ = max (δ , $\bar{\delta}$)

Obviously this unit price discount Δ will also satisfy the m customers C_i . . . C_m who have $\propto^{(i)} = \beta^{(i)}$

Thus all the customers will be satisfied with this unit price discount given by the vendor, ultimately making the compromise at individual JELS level a success. The total inventory stock level available with the vendor at the time of supplying the item to all the n customers should be at least

$$Z^* = \sum_{i=1}^{n} z^{(i)^*}$$

Table-1 Summary of Individual Optimal policies.

	Multiple Customers	**Vendor**
Cost Equation	$C_p^{(i)}(Z^{(i)^*}) =$ $[Z^{(i)^*}\frac{(C_{p2}^{(i)}+C_{v2})\beta}{(\beta+1)} - \left(C_{p1}^{(i)} + C_{p2}^{(i)}\right)A\,\varphi_2(Z^{(i)^*})$ $- C_{p2}^{(i)} Z^{(i)^*} + C_{p2}^{(i)}.\lambda.\left(\frac{1}{k}\right)!]$	$C_v(Z^{(i)^*}) =$ $[Z^{(i)^*}\frac{(C_{p2}^{(i)}+C_{v2})}{(\beta+1)} - (C_{v1} + C_{v2})A\varphi_2$ $(Z^{(i)^*}) - C_{v2}Z^{(i)^*} + C_{v2}\lambda\left(\frac{1}{k}\right)!]$

Economic Lot Size	$Z_p^{(i)^*} = \varphi_1^{-1}[\frac{C_{p2}^{(i)}}{A(C_{p1}^{(i)} + C_{p2}^{(i)})}]$	$Z_v^* = \varphi_1^{-1}[\frac{C_{v2}}{A(C_{v1} + C_{v2})}]$
Minimum Total relevant Cost	$C_p^{(i)^*}(Z_p^{(i)^*}) = [C_{p2}^{(i)} .\lambda .(\frac{1}{k})! - (C_{p1}^{(i)}+C_{p2}^{(i)})A\varphi_2 (Z_p^{(i)^*})]$	$C_v^{(i)}(Z_v^*) = [C_{v2}\, \lambda \left(\frac{1}{k}\right)! - (C_{v1} + C_{v2}) A\varphi_2(Z_v^*)]$

2.4 Illustration through a Numerical Example :

Now we illustrate the model through the following numerical example.

Example : Let the scheduling time t be 1 week.

Let the random lot sizes X_i for each C_i and Y the random lot size of the vendor be independent and identically distributed random variables belonging to Weibull Distribution with the common density function f(x)

Where $f(x) = \begin{cases} Ax^{k-1}e^{-(\frac{x}{\lambda})^k} \\ 0 \quad otherwise \end{cases}$, $0< x < \infty$

With $k = 2, \lambda = 1$ so that $A = \frac{k}{\lambda^k} = 2$

Table-2 , Let us assume that there is no set-up cost. Suppose the parameters used, have been arranged in the following table.

Customer	**$CP_1^{(i)}$**	**$CP_2^{(i)}$**	**CV_1**	**CV_2**	**$\alpha^{(i)}$**	**$\beta^{(i)}$**	**Whether $\alpha^{(i)} = \beta^{(i)}$**	**Whether $\alpha^{(i)} < \beta^{(i)}$**	**Whether $\alpha^{(i)} > \beta^{(i)}$**
C_1	80	20	120	40	0.5	0.625		Yes	
C_2	60	20	120	40	0.5	0.5	Yes		
C_3	56	24	120	40	0.6	0.5			Yes
C_4	76	20	120	40	0.5	0.6		Yes	
C_5	48	16	120	40	0.4	0.4	Yes		
C_6	80	16	120	40	0.4	0.6		Yes	
C_7	108	20	120	40	0.5	0.8		Yes	
C_8	84	28	120	40	0.7	0.7	Yes		

$Z_v^* = \varphi^{-1}[\frac{C_{v2}}{A(C_{v1}+C_{v2})}]$

$\varphi_1 (Z_v^*) = \frac{(Z_v^*)^2}{2} = 0.125$ (2.4.1)

$\therefore Z_v^* = 0.5$ (2.4.2)

for the customer C_1 $(\alpha < \beta)$

$\varphi_1 (Z_p^*) = \frac{C_{p2}}{A(C_{p1} + C_{p2})} = 0.1$

$\therefore Z_p^* = \frac{(Z_p^*)^2}{2} = 0.1$

$Z_p^* = .447213595$ (2.4.3)

$Z^{(i)^*} = \varphi^{-1}[\frac{C_{p2}^{(i)} + C_{v2}}{A(C_{p1}^{(i)}+C_{p2}^{(i)}+C_{v1}+C_{v2})}]$

$\therefore \varphi_1 (Z^{(1)^*}) = 0.115384615$

$\therefore Z^{(1)^*} = \frac{(Z^{(1)^*})^2}{2} = 0.115384615$

$\therefore Z^{(1)^*} = 0.480384461$ (2.4.4)

Here $Z_v^* > Z^{(1)^*} > Z_p^{(1)^*}$ (2.4.5)

for C_2 ($\propto = \beta$)

$\varphi_1(Z_p^*) = 0.125$

$\therefore Z_p^* = 0.5$ (2.4.6)

$\varphi_1(Z^{(2)^*}) = 0.125$

$\therefore Z^{(2)^*} = 0.5$ (2.4.7)

$\therefore Z_v^* = Z^{(2)^*} = Z_p^{(2)^*}$ (2.4.8)

for the customer C_3 ($\propto > \beta$)

$\varphi_1(Z_p^*) = \frac{C_{p2}}{A(C_{p1} + C_{p2})} = 0.15$

$\therefore (Z_p^{(3)^*}) = 0.547722557$ (2.4.9)

$\varphi_1(Z^{(3)^*}) = \frac{C_{p2}^{(3)} + C_{v2}}{A(C_{p1}^{(3)} + C_{p2}^{(3)} + C_{v1} + C_{v2})} = 0.133333333$

$\therefore Z^{(3)^*} = 0.516397779$ (2.4.10)

Here, $Z_v^* < Z^{(3)^*} < Z_p^{(3)^*}$ (2.4.11)

for the customer C_4 ($\propto < \beta$)

$\varphi_1(Z_p^*) = 0.104166666$

$\therefore Z_p^* = .456435463$ (2.4.12)

$\varphi_1(Z^{(4)^*}) = 0.1171875$

$\therefore Z^{(4)^*} = .484122918$ (2.4.13)

Here, $Z_v^* > Z^{(4)^*} > Z_p^{(4)^*}$

For the customer C_5 ($\propto = \beta$)

$\varphi_1(Z_p^*) = 0.125$ (2.4.14)

$\therefore Z_p^* = 0.5$ (2.4.15)

$Z^{(5)^*} = 0.125$ (2.4.16)

$Z^{(5)^*} = 0.5$ (2.4.17)

Here, $Z_v^* = Z^{(5)^*} = Z_p^{(5)^*}$

For the customer C_6 ($\propto < \beta$)

$\therefore \varphi_1(Z_p^*) = .083333333$

$\therefore Z_p^* = 0.40824829$ (2.4.18)

$\varphi_1(Z^{(6)^*}) = 0.109375$

$Z^{(6)^*} = .467707173$ (2.4.19)

Here, $Z_v^* > Z^{(6)^*} > Z_p^{(6)^*}$ (2.4.20)

For the customer C_7 ($\propto < \beta$)

$\therefore \varphi_1(Z_p^*) = 0.078125$

$Z_p^* = .395284707$ (2.4.21)

$\therefore \varphi_1(Z^{(7)^*}) = 0.104166666$

$Z^{(7)^*} = .456435464$ (2.4.22)

Here, $Z_v^* > Z^{(7)^*} > Z_p^{(7)^*}$

For the customer C_8 ($\propto = \beta$)

$\therefore \varphi_1(Z_p^*) = 0.125$ (2.4.23)

$Z_p^* = 0.5$ (2.4.24)

$\therefore \varphi_1(Z^{(8)^*}) = 0.125$

$Z^{(8)^*} = 0.5$ (2.4.25)

Here, $Z_v^* = Z^{(8)^*} = Z_p^{(8)^*}$ (2.4.26)

Let us find the values C_1, C_4, C_6, C_7

$C_v(Z_p^{(i)^*}) = [$

$\frac{Z_p^{(i)^*} C_{p2}^i}{\beta} - (C_{v1} + C_{v2})A\varphi_2\left(Z_p^{(i)^*}\right) - C_{v2}Z_p^{(i)^*} + C_{v2}\,\lambda\,(\frac{1}{k})!$

]

∴ for customer C_1

$\varphi_2\ (Z_p^{(1)^*}) = \frac{(Z_p^{(1)^*})^3}{3} = 0.029814239$

$[(\frac{1}{2})!$=.760326419 According to sterling function] (2.4.27)

$C_v(Z_p^{(i)^*})$=17.29479152

$\therefore\ \varphi_2(Z^{(1)^*}) = \frac{(Z^{(1)^*})^3}{3} = 0.03695265$

$C_v(Z^{(1)^*})=[\frac{Z^{(1)^*}}{3}\ (C_{p2}^1 + C_{v2})$- $(C_{v1} + C_{v2})$

$A\varphi_2(Z^{(1)^*}) - C_{v2}Z^{(1)^*} + C_{v2}.\lambda.\left(\frac{1}{k}\right)!]$

= 17.11010273 (2.4.28)

$\therefore\ C_v(Z_p^{(1)^*})$ - $C_v(Z^{(1)^*}) > 0$

for customer C_4

$\varphi_2(Z_p^{(4)^*}) = \frac{(Z_p^{(4)^*})^3}{3} = 0.031696906$

$C_v(Z_p^{(4)^*}) = 17.22714375$ (2.4.29)

$\varphi_2(Z^{(4)^*}) = \frac{(Z^{(4)^*})^3}{3} = 0.037822102$

$C_v(Z^{(4)^*})$=

[

$\frac{Z^{(4)^*}(C_{p2}^4+C_{v2})}{\beta+1} - (C_{v1}+C_{v2})A\varphi_2\left(Z^{(4)^*}\right) - C_{v2}\left(Z^{(4)^*}\right) + C_{v2}\lambda.\left(\frac{1}{k}\right)!$

]

= 17.09967683 (2.4.30)

$C_v\ (Z_p^{(4)^*})$ - $C_v(Z^{(4)^*}) > 0$

for customer C_6

$\varphi_2(Z_p^{(6)^*}) = .02268046$

$C_v(Z_p^{(6)^*})$=

$\frac{Z_p^{(6)^*}C_{p2}^6}{\beta} - (C_{v1}+C_{v2})A\varphi_2\left(Z_p^{(6)^*}\right) - C_{v2}Z_p^{(6)^*} + C_{v2}.\lambda.\left(\frac{1}{k}\right)!$

=17.71199903 (2.4.31)

$\varphi_2\ (Z^{(6)^*}) = \frac{(Z^{(6)^*})^3}{3} = .034103647$

$C_v(Z^{(6)^*})=\frac{Z^{(6)^*}(C_{p2}^6+C_{v2})}{1+\beta}$ - $(C_{v1} + C_{v2})$

$A\varphi_2(Z^{(6)^*})$ - $C_{v2}\ Z^{(6)^*} + C_{v2}\lambda.\left(\frac{1}{k}\right)!$

=17.16135386 (2.4.32)

$C_v\ (Z_p^{(6)^*})$ -$C_v(Z^{(6)^*})>0$

by (2.4.31) & (2.4.32)

For customer C_7

$\varphi_2\ (Z_p^{(7)^*}) = \frac{(Z_p^{(7)^*})^3}{3}$ =.020587745

$C_v\ (Z_p^{(7)^*})=\frac{Z^{(7)^*}C_{p2}^7}{\beta}$ - $(C_{v1} + C_{v2})\ A\varphi_2\ (Z_p^{(7)^*})$

- $C_{v2}\ (Z_p^{(7)^*}) + C_{v2}\lambda.\left(\frac{1}{k}\right)!$

=17.89570776 (2.4.33)

$\varphi_2\ (Z^{(7)^*}) = \frac{(Z^{(7)^*})^3}{3} = .031696907$

$C_v(Z^{(7)^*})=\frac{Z^{(7)^*}(C_{p2}^7+C_{v2})}{1+\beta}$-$(C_{v1} + C_{v2})$

$A\varphi_2(Z^{(7)^*})$ - $C_{v2}\ Z^{(7)^*} + C_{v2}\lambda.\left(\frac{1}{k}\right)!$

=17.22714343 (2.4.34)

$C_v\ (Z_p^{(7)^*})$ -$C_v(Z^{(7)^*})>0$

by (2.4.33) & (2.4.34)

Let us calculate

ACA$_v(Z_p^{(1)^*} \to Z^{(1)^*})$ & AC$P_{C_i}(Z_p^{(1)^*} \to Z^{(1)^*})$

for i = 1,2,3,4,5,6,7,8

ACA$_v(Z_p^{(1)^*} \to Z^{(1)^*})$ = $C_v(Z_p^{(1)^*})$ - $C_v(Z^{(1)^*})$

= 0.18468879 (2.4.35)

To calculate ACA$_{c1}$ $(Z_p^{(1)^*} \to Z^{(1)^*})$, we have to calculate $C_p^{(1)}(Z^{(1)^*})$ and $C_p^{(1)}(Z_p^{(1)^*})$

$C_p^{(1)}(Z^{(1)^*})=[\frac{z^{(1)^*}(c_{p_2}^{(1)}+c_{v2})\beta}{1+\beta}$ - $(C_{p1}^{(1)}+C_{p2}^{(1)})$ $A\varphi_2(Z^{(1)^*}) - C_{p2}^{(1)}Z^{(1)^*} + C_{p2}^{(1)}\,\lambda.\left(\frac{1}{k}\right)!]$

=9.29410441 (2.4.36)

$C_p^{(1)}(Z_p^{(1)^*})$ = [$C_{p2}^{1}\,\lambda.\left(\frac{1}{k}\right)!$ - $(C_{p1}^{(1)}+C_{p2}^{(1)})$ $A\varphi_2 Z_p^{(1)^*})]$

=9.24368058 (2.4.37)

$C_p^{(1)}(Z^{(1)^*})$- $C_p^{(1)}(Z_p^{(1)^*})$ =.05042383 (2.4.38)

Hence

ACA$_v(Z_p^{(1)^*} \to Z^{(1)^*})$> ACP$_{c1}(Z_p^{(1)^*} \to Z^{(1)^*})$

JACA$(Z_p^{(1)^*} \to Z^{(1)^*})$=0.13426496

(2.4.39)

JACA$(Z_p^{(1)^*} \to Z^{(1)^*})$

$=[\frac{C_{p2}^{(1)}Z_p^{(1)^*}}{\beta} - C_{v2}Z_p^{(1)^*} + (C_{p1}^{1} + C_{p2}^{1} + C_{v1} + C_{v2})(A(\varphi_2(Z^{(1)^*}) - \varphi_2(Z_p^{(1)^*}))]$

=0.13426496 (2.4.40)

Thus the results of (2.4.39) & (2.4.40) tally optimum discount to the customer

$C_1 = \frac{1}{2*\lambda.\left(\frac{1}{k}\right)!}$ JACA$(Z_p^{(1)^*} \to Z^{(1)^*})$

=0.088294288 (2.4.41)

For customer C_4

$C_p^{(4)}(Z^{(4)^*})=[\frac{z^{(4)^*}(c_{p_2}^{(4)}+c_{v2})\beta}{1+\beta}$ - $(C_{p1}^{(4)}+C_{p2}^{(4)})$ $A\varphi_2(Z^{(4)^*})$ - $C_{p2}^{(4)}Z^{(4)^*} + C_{p2}^{(4)}\,\lambda.\left(\frac{1}{k}\right)!]$

=9.154992091 (2.4.42)

$C_p^{(4)}(Z_p^{(4)^*})$ = [$C_{p2}^{4}\,\lambda.\left(\frac{1}{k}\right)!$ - $(C_{p1}^{(4)}+C_{p2}^{(4)})$ $A\varphi_2 Z_p^{(4)^*})]$

= 9.12088486 (2.4.43)

ACP$c_4(Z_p^{(4)^*} \to Z^{(4)^*})$= $C_p^{(4)}(Z^{(4)^*}) - C_p^{(4)}(Z_p^{(4)^*})$

= .034107231 (2.4.44)

ACA$_v(Z_p^{(4)^*} \to Z^{(4)^*})$=0.12746692 (2.4.45)

Hence,

ACA$_v(Z_p^{(4)^*} \to Z^{(4)^*})$ > ACP$c_4(Z_p^{(4)^*} \to Z^{(4)^*})$

JACA$(Z_p^{(4)^*} \to Z^{(4)^*})$=.093359689 (2.4.46)

JACA$(Z_p^{(4)^*} \to Z^{(4)^*})$

$=[\frac{C_{p2}^{(4)}Z_p^{(4)^*}}{\beta} - C_{v2}Z_p^{(4)^*} + (C_{p1}^{4} + C_{p2}^{4} + C_{v1} + C_{v2})(A(\varphi_2(Z^{(4)^*}) - \varphi_2(Z_p^{(4)^*}))]$

=.093197265 (2.4.47)

Thus (2.4.46) & (2.4.47) agree with each other

∴ Optimum discount to C_4

$=\frac{1}{2.\lambda\left(\frac{1}{k}\right)!}$JACA$(Z_p^{(4)^*} \to Z^{(4)^*})$

= .035430171 (2.4.48)

For customer C_6

$C_p^{(6)} Z^{(6)^*}$ = $[\frac{z^{(6)^*}(C_{p_2}^{(6)}+C_{v2})\beta}{1+\beta}$ - $(C_{p1}^{(6)} + C_{p2}^{(6)})$

$A\varphi_2(Z^{(6)^*})$ - $C_{p2}^{(6)} Z^{(6)^*}$ + $C_{p2}^{(6)}\ \lambda.\left(\frac{1}{k}\right)!]$

=7.955858345 (2.4.49)

$C_p^{(6)}(Z^{(6)^*})$=$[C_{p2}^{6}.\lambda.\left(\frac{1}{k}\right)!$-$(C_{p1}^{6} + C_{p2}^{6})$

$A\varphi_2(Z_p^{(6)^*})]$

=7.810574384 (2.4.50)

ACP$c_6(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$= $c_p^{(6)}(Z^{(6)^*}) - c_p^{(6)}(Z_p^{(6)^*})$

= 0.145283961 (2.4.51)

$ACA_v(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$=0.55064517 (2.4.52)

Hence ,

$ACA_v(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$ >ACP$c_6(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$

JACA$(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$=.405361209 (2.4.53)

By formula, JACA$(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$

=$[\frac{c_{p2}^{(6)} z_p^{(6)^*}}{\beta} - C_{v2}Z_p^{(6)^*}$+$(C_{p1}^{6} + C_{p2}^{6}+C_{v1} + C_{v2})$

$(A(\varphi_2(Z^{(6)^*}) - \varphi_2(Z_p^{(6)^*}))]$

=0.40536121 (2.4.54)

Thus, (2.4.53) & (2.4.54) tally with each other.

∴ Optimum discount to

$C_6 = \frac{1}{2.\lambda\left(\frac{1}{k}\right)!}$ JACA $(Z_p^{(6)^*} \rightarrow Z^{(6)^*})$

= .266570515 (2.4.55)

For customer C_7

$C_p^{(7)}(Z^{(7)^*})$= $[\frac{z^{(7)^*}(C_{p_2}^{(7)}+C_{v2})\beta}{1+\beta}$ - $(C_{p1}^{(7)} + C_{p2}^{(7)})$

$A\varphi_2(Z^{(7)^*})$ - $C_{p2}^{(7)} Z^{(7)^*}$ + $C_{p2}^{(7)}\ \lambda.\left(\frac{1}{k}\right)!]$

= 10.13502328 (2.4.56)

$C_p^{(7)}(Z^{(7)^*})$ = $[C_{p2}^{(7)}\ \lambda.\left(\frac{1}{k}\right)!$-$(C_{p1}^{(7)} + C_{p2}^{(7)})$

$A\varphi_2(Z^{(7)^*})]$

= 9.93606566 (2.4.57)

ACP$c_7(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$= $c_p^{(7)}(Z^{(7)^*}) - c_p^{(7)}(Z_p^{(7)^*})$

= 0.19895762 (2.4.58)

$ACA_v(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$=0.66856433 (2.4.59)

Hence,

$ACA_v(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$ >ACP$c_7(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$

JACA$(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$=0.46960671 (2.4.60)

Again by formula , JACA$(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$

= $[\frac{c_{p2}^{(7)} z_p^{(7)^*}}{\beta} - C_{v2}Z_p^{(7)^*}$+ $(C_{p1}^{(7)} + C_{p2}^{(7)}$ +

$C_{v1} + C_{v2}$)A$\{(\varphi_2(Z^{(7)^*}) - \varphi_2(Z_p^{(7)^*})\}]$

=0.469606707 (2.4.61)

Thus the result (2.4.60) is in agreement with the result (2.4.61)

∴ for customer C_7

Optimum discount= $\frac{1}{2.\lambda\left(\frac{1}{k}\right)!}$ JACA $(Z_p^{(7)^*} \rightarrow Z^{(7)^*})$

=0.308819143 (2.4.62)

∴δ=max{0.088294288,.035430171, .266570515, 0.308819143}

=0.308819143 (2.4.63)

Third Category : $\propto^{(i)} > \beta^{(i)}$

C_3 is the only member belonging to this category for the customer C_3

$C_v(Z_p^{(3)^*})=$

$$\left[\frac{z_{p2}^{(3)^*} c_{p2}^{(3)}}{\beta} (C_{v1}+C_{v2})A\varphi_2\left(Z_p^{(3)^*}\right) - C_{v2}Z_p^{(3)^*} + C_{v2}.\lambda.\left(\frac{1}{k}\right)! \right]$$

$\varphi_2(Z_p^{(3)^*})=\frac{(Z_p^{(3)^*})^3}{3} = \frac{(0.547722557)^3}{3} = 0.054772255$

$\varphi_2(Z^{(3)^*}) = \frac{(Z^{(3)^*})^3}{3} = 0.045902024$

$\therefore C_v(Z_p^{(3)^*})$=17.26771562 (2.4.64)

$C_v(Z^{(3)^*})=[\frac{z^{(3)^*}(c_{P2}^{(3)}+C_{v2})}{1+\beta}-(C_{v1}+C_{v2})$

$A\varphi_2(Z^{(3)^*}) - C_{v2} Z^{(3)^*} + C_{v2}\lambda.\left(\frac{1}{k}\right)!]$

=17.10146982 (2.4.65)

ACA$_v(Z_p^{(3)^*} \to Z^{(3)^*})$=0.1662458 (2.4.66)

$C_p^{(3)}(Z^{(3)^*})= [\frac{z^{(3)^*}(c_{P2}^{(3)}+C_{v2})\beta}{1+\beta} - (C_{p1}^{(3)} + C_{p2}^{(3)})$

$A\varphi_2(Z^{(3)^*}) - C_{p2}^{(3)} Z^{(3)^*} + C_{p2}^{(3)}\ \lambda.\left(\frac{1}{k}\right)!]$

=9.526449472 (2.4.67)

$C_p^{(3)}(Z_p^{(3)^*}) = [\ C_{p2}^{(3)}\ \lambda.\left(\frac{1}{k}\right)! - (C_{p1}^{(3)} + C_{p2}^{(3)})$

$A\varphi_2 Z_p^{(3)^*})]$

= 9.484273256 (2.4.68)

ACP$c_3(Z_p^{(3)^*} \to Z^{(3)^*})= C_p^{(3)}\left(Z^{(3)^*}\right) - C_p^{(3)}(Z_p^{(3)^*})$

= 0.042176216 (2.4.69)

Hence,

ACA$_v(Z_p^{(3)^*} \to Z^{(3)^*})$>ACP$c_3$ $(Z_p^{(3)^*} \to Z^{(3)^*})$

$\therefore$ JACA$(Z_p^{(3)^*} \to Z^{(3)^*})$=0.124069584 (2.4.70)

By formula

JACA$(Z_p^{(3)^*} \to Z^{(3)^*})$

$=[\frac{c_{p2}^{(3)} Z_p^{(3)^*}}{\beta} - C_{v2}Z_p^{(3)^*} + (C_{p1}^{(3)} + C_{p2}^{(3)} + C_{v1} + C_{v2})$

$(A(\varphi_2(Z^{(3)^*}) - \varphi_2(Z_p^{(3)^*}))]$

= 0.12406958 (2.4.71)

Thus the result (2.4.70) is agreement with (2.4.71)

Therefore the optimum value of discount offered to c_3

$= \frac{1}{2.\lambda\left(\frac{1}{k}\right)!}$ JACA $(Z_p^{(3)^*} \to Z^{(3)^*})$

= 0.081589681 (2..4.72)

As c_3 is the only customer belonging to third category

$\therefore \bar{\delta}$=0.081589681 (2.4.73)

From (2.4.72) & (2.4.73) the optimum value of the uniform discount given to all the eight customers is given by

$\therefore \Delta = max(\delta, \bar{\delta})$=max(0.308819143, 0.0815896801)

=0.308819143 (2.4.74)

The total inventory stock level available with the vendor at the time of supplying the item to all the eight customers should be at least

$$Z^* = \sum_{i=1}^{8} Z^{(i)*}$$

$$= Z^{(1)*} + Z^{(2)*} + Z^{(3)*} + Z^{(4)*} + Z^{(5)*} + Z^{(6)*} + Z^{(7)*} + Z^{(8)*}$$

$$= 3.905047795 \qquad (2.4.75)$$

5. Conclusion

In this paper, the buyer-vendor area of the supply chain management problem discussed. Here mainly focused on the Joint Economic Lot Size for the buyer and vendor model. There are many models which recently extended Banerjee's JELS. Banerjee's (1986), showed that his model worked for a single product, single buyer and single vendor. He showed great savings with his model. Here a model developed for Single vendor and multiple buyer situations using Weibulls Distribution.

In this model a detailed analysis has been made to show how inventory related costs vary through closer interaction between the vendor and the customer. The unit price and the order quantity etc. are settled by negotiation between both the parties to minimize the total relevant costs. If JELS is adopted by both, the gain or loss are to be shared reasonably between them so that both will come to a mutual compromise. JELS model not only minimize the total relevant cost of the system but also searches a common lot size with no loss to both. In this model the set up cost is assumed to be zero. The effect of this JELS model can be verified in various other situation with demand satisfying different continuous probability distributions. The demand of the customer and stock level of the vendor are non-negative quantities.

6. References

[1] Monahan, J. P. (1984), A Quantity Discount Pricing Model to Increase Vendor Profits, Management Science 30, 720-726.

[2] Lee, H. L. and Rosenblatt, M. J. (1986), A Generalized Quantity Discount Pricing Model to Increase Suppliers Profits, Management Science, 32 1177-1185

[3] Banerjee A. (1986 a), A Joint Economic Lot size Model for purchaser and vendor, Decision science 17, 292-311.

[4] Jogelkar, P. N. (1988), Comments on A Quantity Discount pricing Model to Increase Vendor Profits, Management Science 31, 1391-1398.

[5] Goyal, S.K. (1988), A joint economic lot size model for purchaser and vendor: a comment. *Decision Sciences*, 19, 236-241.

[6] Goyal, S. K and Gupta, Y. P. (1989), Integrated Inventory Models: the Buyer-vendor co-ordination, European Journal of operations research 41, 261-269.

[7] Vishwanathan S. (1998), Optimal strategy for the integrated vendor-buyer inventory model. European Journal of Operational Research; 105: 38-42.

[8] Chatterjee, A. K., Ravi, R. (1991), Joint Economic Lot Size model with delivery in Sub-batches OPSEARCH, 28, 119-124.

[9] Agrawal, A. K., Raju D. A. (1996), Improved Joint Economic Lot Size model for a purchaser and a vendor. Department of

Mechanical Engineering, Institute of Technology, Benras Hindu University, Varanasi, India.

[10] Thomas, D. J., Griffin, D. J. (1996). Coordinated supply chain management. E*uropean Journal of Operational Research, 94*, 1-15.

[11] Narasimhan and Carter (1998), Integrating environmental management and supply chain strategies, Volume: 14, Wiley InterScience, Pages: 1-19.

[12] Affisco J.F. , Paknejad M.J., and Nasri F., (2002), Quality improvement and setup reproduction in the Joint Economic Lot Size model. European Journal of Operations Research , 142, 497-508.

[13] Pourakbar, M., Farahani, Z.R. & Asgari, N. (2007), A Joint Economic Lot Size model for an Integrated Supply network using genetic algorithm. Applied Mathematics and Computation , 189(1). 583-596.

[14] Ben-Daya M., Darwish M. and Ertogral K.,(2008), The Joint Economic Lot Sizing problem. A review and extensions. European Journal of Operational Research, 185/2, 726-742.

[15] Pan J.C.-H & Yang M. –F.(2008), Integrated inventory models with fuzzy annual demand and fuzzy production rate in a Supply Chain. International Journal of Production Research 46(3), 753-770.